Odrediti koji član razvoja binoma $\left(\frac{3}{4}\sqrt[3]{a^2}+\frac{2}{3}\sqrt{a}\right)^{12}$ sadrži a^7.

Rj. $\left(\frac{3}{4}\sqrt[3]{a^2}+\frac{2}{3}\sqrt{a}\right)^{12}=\sum_{k=0}^{12}\binom{12}{k}\left(\frac{3}{4}\sqrt[3]{a^2}\right)^{12-k}\cdot\left(\frac{2}{3}\sqrt{a}\right)^{k}=$

$$=\sum_{k=0}^{12}\binom{12}{k}\left(\frac{3}{4}\right)^{12-k}a^{\frac{2(12-k)}{3}}\cdot\left(\frac{2}{3}\right)^{k}\cdot a^{\frac{k}{2}}=\sum_{k=0}^{12}\binom{12}{k}\left(\frac{3}{4}\right)^{12-k}\cdot\left(\frac{2}{3}\right)^{k}\cdot a^{8-\frac{2k}{3}+\frac{k}{2}}$$

$$=\sum_{k=0}^{12}\binom{12}{k}\left(\frac{3}{4}\right)^{12-k}\left(\frac{2}{3}\right)^{k}a^{8-\frac{k}{6}}$$

Tražimo član koji sadrži a^7.

$8-\frac{k}{6}=7 \quad /\cdot 6$

$48-k=42$

$k=6$

Sedmi član u razvoju binoma sadrži a^7.

Riješiti matričnu jednačinu $(A+3I)(X-I)=B$, ako je $A=\begin{bmatrix}-2&5&-2\\2&8&0\\-1&-5&-2\end{bmatrix}$, $B=\begin{bmatrix}-3&21&1\\2&50&-2\\1&-22&0\end{bmatrix}$ i I jedinična matrica.

Rj. $(A+3I)(X-I)=B \quad /\cdot(A+3I)^{-1}$ sa lijeve strane

$(A+3I)^{-1}(A+3I)(X-I)=(A+3I)^{-1}\cdot B$

$X-I=(A+3I)^{-1}\cdot B$

$X=(A+3I)^{-1}\cdot B+I$

$C^{-1}=\frac{1}{\det C}C_{kof}^{T}$

$\det C=\begin{vmatrix}1&5&-2\\2&11&0\\-1&-5&1\end{vmatrix}\ \underline{\underline{I_R+III_R}}$

$=\begin{vmatrix}0&0&-1\\2&11&0\\-1&-5&1\end{vmatrix}=(-1)\begin{vmatrix}2&11\\-1&-5\end{vmatrix}=-1$

$C=A+3I=\begin{bmatrix}-2&5&-2\\2&8&0\\-1&-5&-2\end{bmatrix}+\begin{bmatrix}3&0&0\\0&3&0\\0&0&3\end{bmatrix}=\begin{bmatrix}1&5&-2\\2&11&0\\-1&-5&1\end{bmatrix}$

$C_{11}=(-1)^2\begin{vmatrix}11&0\\-5&1\end{vmatrix}=11$

$C_{12}=(-1)^3\begin{vmatrix}2&0\\-1&1\end{vmatrix}=-2$

$C_{13}=(-1)^4\begin{vmatrix}2&11\\-1&-5\end{vmatrix}=1$

$C_{21}=(-1)^3\begin{vmatrix}5&-2\\-5&1\end{vmatrix}=5$

$C_{22}=(-1)^4\begin{vmatrix}1&-2\\-1&1\end{vmatrix}=-1$

$C_{23}=(-1)^5\begin{vmatrix}1&5\\-1&-5\end{vmatrix}=0$

$C_{31}=(-1)^4\begin{vmatrix}5&-2\\11&0\end{vmatrix}=22$

$C_{32}=(-1)^5\begin{vmatrix}1&-2\\2&0\end{vmatrix}=-4$

$C_{33}=(-1)^6\begin{vmatrix}1&5\\2&11\end{vmatrix}=1$

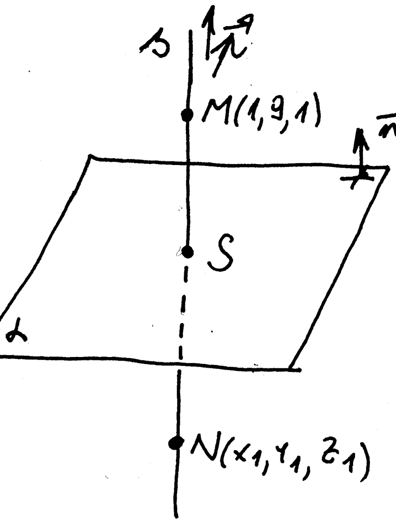
$C_{kof}^{T}=\begin{bmatrix}11&5&22\\-2&-1&-4\\1&0&1\end{bmatrix}$

$C^{-1}=\begin{bmatrix}-11&-5&-22\\2&1&4\\-1&0&-1\end{bmatrix}$

$C^{-1}\cdot B=\begin{bmatrix}-11&-5&-22\\2&1&4\\-1&0&-1\end{bmatrix}\cdot\begin{bmatrix}-3&21&1\\2&50&-2\\1&-22&0\end{bmatrix}=\begin{bmatrix}1&3&-1\\0&4&0\\2&1&-1\end{bmatrix}$

\# Odrediti tačku koja je simetrična tački $M(1,9,1)$ u odnosu na ravan $\alpha: 2x+y+3z=0$.

Rj.



$M(1,9,1)$

$\alpha: 2x+y+3z=0$

$M \notin \alpha$

$N=?$ $\qquad |\overrightarrow{MS}| = |\overrightarrow{NS}|$

Da bismo odredili tačku N prvo ćemo postaviti pravu s koja je okomita na α i uz pomoć te prave naći tačku S.

$\vec{n}=(2,1,3)$

$\vec{p} \parallel \vec{n} \Rightarrow$ mogu uzeti $\vec{p}=(2,1,3)$

$s: \dfrac{x-1}{2} = \dfrac{y-9}{1} = \dfrac{z-1}{3}$ $(=t)$

$$s: \begin{cases} x = 2t+1 \\ y = t+9 \\ z = 3t+1 \end{cases}$$

$x-1=2t$

$y-9=t$

$z-1=3t$

$2x+y+3z=0$

$2(2t+1)+(t+9)+3(3t+1)=0$

$\underline{\underline{4t}}+2+\underline{\underline{t}}+9+\underline{\underline{9t}}+3=0$

$14t=-14$

$t=-1$

Tačka presjeka prave s i ravni α je $S(-1,8,-2)$

$M(1,9,1)$

$S(-1,8,-2)$ $\qquad \overrightarrow{MS}=(-2,-1,-3)$

$N(2t+1, t+9, 3t+1)$

$S(-1, 8, -2)$ $\qquad \overrightarrow{NS}=(-2t-2, -t-1, -3t-3)$

$|\overrightarrow{MS}| = \sqrt{4+1+9} = \sqrt{14}$

$|\overrightarrow{NS}| = \sqrt{(-2t-2)^2+(-t-1)^2+(-3t-3)^2}$

$(-2t-2)^2 = 4t^2+8t+4$

$(-t+1) = t^2+2t+1$

$(-3t-3)^2 = 9t^2+18t+9$

$14t^2+28t+14$

$|\overrightarrow{MS}| = |\overrightarrow{NS}|$

$14t^2+28t+\underline{14} = \underline{14} \quad /:14$

$t^2+2t=0$

$t(t+2)=0$

$t=0$ ili $t=-2$

$N(-3,7,-5)$

tražena tačka

\# Ispitati f-ju i nacrtati joj grafik $y = \frac{3x}{1+x^3}$.

Rj. definiciono područje

$1+x^3 \neq 0$

$x^3 \neq -1$

$x \neq -1$

$\mathcal{D}: x \in (-\infty, -1) \cup (-1, +\infty)$

parnost, neparnost, periodičnost

$f(-x) = \frac{3\cdot(-x)}{1+(-x^3)} = -\frac{3x}{1-x^3}$ f-ja nije ni parna ni neparna

f-ja nije periodična

nule, presjek sa y-osom, znak f-je

$Y = 0$

$\frac{3x}{1+x^3} = 0$

$x = 0$

(0,0) je nula f-je i presjek sa Y-osom

x	$(-\infty,-1)$	$(-1,0)$	$(0,+\infty)$
$3x$	−	−	+
$1+x^3$	−	+	+
Y	+	−	+

znak f-je

ponašanje na krajevima intervala definisanosti i asimptote

za vrijednost $x = -1$ f-ja ima prekid

$\lim_{x\to -1-0} f(x) = \lim_{x\to -1-0} \frac{3x}{1+x^3} = \frac{3(-1-0)}{1+(-1-0)^3} = \frac{3(-1-0)}{1-1-0} = \frac{-3-0}{-0} = +\infty \Rightarrow x = -1$ je V.A.

$\lim_{x\to -1+0} f(x) = \lim_{x\to -1+0} \frac{3x}{1+x^3} = \frac{3(-1+0)}{1+(-1+0)^3} = \frac{-3+0}{1-1+0} = \frac{-3+0}{+0} = -\infty \Rightarrow x = -1$ je V.A.

$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{3x \,:x}{1+x^3 \,:x} = \lim_{x\to -\infty} \frac{3}{\underbrace{\frac{1}{x}}_{\to 0} + x^2} = 0 \Rightarrow Y = 0$ je H.A.

$\lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{3}{\underbrace{\frac{1}{x}}_{\to 0} + x^2} = 0 \Rightarrow Y = 0$ je H.A. f-ja nema K.A.

rast i opadanje

$y' = \left(\frac{3x}{1+x^3}\right)' = 3\cdot\frac{1\cdot(1+x^3) - x\cdot 3x^2}{(1+x^3)^2} = 3\,\frac{1+x^3-3x^3}{(1+x^3)^2}$

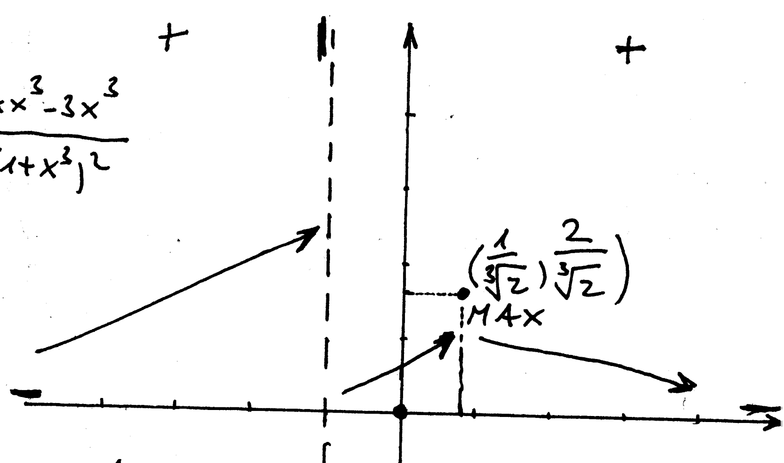
$y' = 3\cdot\frac{1-2x^3}{(1+x^3)^2}$

$y' = 0$ akko $1-2x^3 = 0$

$2x^3 = 1$

$x^3 = \frac{1}{2}$

$x = \frac{1}{\sqrt[3]{2}} \approx 0{,}8$



$-1 \qquad \frac{1}{\sqrt[3]{2}}$

prekidi Y + nule Y'

x	$(-\infty,-1)$	$(-1,\frac{1}{\sqrt[3]{2}})$	$(\frac{1}{\sqrt[3]{2}},+\infty)$
y'	+	+	−
y	↗	↗	↘

MAX

ekstremi f-je

Na osnovu tabele

$f\left(\frac{1}{\sqrt[3]{2}}\right) = \frac{\frac{3}{\sqrt[3]{2}}}{1+\frac{1}{2}} = \frac{\frac{3}{\sqrt[3]{2}}}{\frac{3}{2}} = \frac{2}{\sqrt[3]{2}} \approx 1{,}6$

$\Rightarrow \left(\frac{1}{\sqrt[3]{2}}, \frac{2}{\sqrt[3]{2}}\right)$ je tačka maksimuma prevojne tačke i intervali konveksnosti i konkavnosti

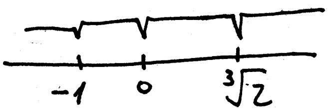
$$y' = 3\cdot\frac{1-2x^3}{(1+x^3)^2}, \quad y'' = 3\cdot\frac{-6x^2\cdot(1+x^3)^{\cancel{2}} - (1-2x^3)\cdot 2\cancel{(1+x^3)}\cdot 3x^2}{(1+x^3)^3\cdot\cancel{(1+x^3)}} =$$

$$= 3\cdot\frac{-6x^2-6x^5-6x^2+12x^5}{(1+x^3)^3} = 3\cdot\frac{6x^5-12x^2}{(1+x^3)^3}$$

$$y'' = 18\cdot\frac{x^5-2x^2}{(1+x^3)^3} = \frac{18x^2(x^3-2)}{(1+x^3)^3}$$

$y''=0$ akko $x=0$ ili $x^3-2=0$

$x_1=0$ $\quad x_2=\sqrt[3]{2}\approx 1{,}3$



x	$(-\infty,-1)$	$(-1,0)$	$(0,\sqrt[3]{2})$	$(\sqrt[3]{2},+\infty)$
y''	+	−	−	+
y	∪	∩	∩	∪

P_0T_0

$$f(\sqrt[3]{2}) = \frac{3\sqrt[3]{2}}{1+2} = \sqrt[3]{2}$$

$(\sqrt[3]{2}, \sqrt[3]{2})$ je prevojna tačka

grafik